MATH 1A - HOW TO DERIVE THE FORMULA FOR THE DERIVATIVE OF ARCCOS(X) - WRITE-UP

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Please refer to the other handout 'Arccos' for more details! This handout is just about how to **write up** your solution for the problem below. The other handout gives way more details!

Problem: Show that the derivative of $y = \cos^{-1}(x)$ is $y' = \frac{-1}{\sqrt{1-x^2}}$

SOLUTION: Let $y = cos^{-1}(x)$, so $\cos(y) = x$, hence, using implicit differentiation:

$$\begin{aligned} (\cos(y))' &= (x)' \\ y' \cdot (-\sin(y)) &= 1 \\ y' &= \frac{-1}{\sin(y)} \\ y' &= \frac{-1}{\sin(\cos^{-1}(x))} \end{aligned}$$

For the rest, you can **EITHER** choose the geometric way or the algebraic way (**DO NOT** do both, it's a complete waste of time!!!)

0.1. **Geometric way.** Let $\theta = \cos^{-1}(x)$, so $x = \cos(\theta) = \frac{AC}{BC}$ in the picture below:

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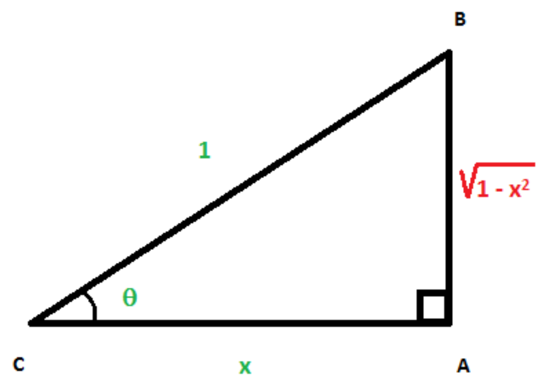

1A/Triangle.png

Then $\sin(\cos^{-1}(x)) = \sin(\theta) = \frac{AB}{BC}$, but $BC = 1$, and, by the Pythagorean theorem:

$$\begin{aligned} BC^2 &= AB^2 + AC^2 \\ AB^2 &= BC^2 - AC^2 \\ AB^2 &= 1 - x^2 \\ AB &= \sqrt{1-x^2} \end{aligned}$$

And now we're done, because: $\sin(\cos^{-1}(x)) = \frac{AB}{BC} = AB = \sqrt{1-x^2}$, and hence:

$$y' = \frac{-1}{\sin(\cos^{-1}(x))} = \frac{-1}{\sqrt{1-x^2}}$$

0.2. **Algebraic way.** From $\boxed{\sin^2(x) + \cos^2(x) = 1}$, with $\cos^{-1}(x)$ instead of x, we get:

$$\begin{aligned} \sin^2(\cos^{-1}(x)) + \cos^2(\cos^{-1}(x)) &= 1 \\ \sin^2(\cos^{-1}(x)) + x^2 &= 1 \\ \sin^2(\cos^{-1}(x)) &= 1 - x^2 \\ \sin(\cos^{-1}(x)) &= \pm\sqrt{1-x^2} \end{aligned}$$

Now $\cos^{-1}(x)$ has range $[0, \pi]$, so $\sin(\cos^{-1}(x)) \geq 0$ it follows that $\sin(\cos^{-1}(x)) = \sqrt{1-x^2}$.

So, we get:

$$y' = \frac{-1}{\sin(\cos^{-1}(x))} = \frac{-1}{\sqrt{1-x^2}}$$